

|   |  |   |  |           |                |            |                 |                          |                         |               |                  |                 |  |
|---|--|---|--|-----------|----------------|------------|-----------------|--------------------------|-------------------------|---------------|------------------|-----------------|--|
| Course title  | Mathematical Methods for Economic Analysis |   |  |           |                |            |                 |                          |                         |               | ECTS code        | 14.3.EE.FL.3511 |  |
|   |  |   |  |           |                |            |                 |                          |                         |               | ECTS credits     | 5               |  |
|   |  |   |  |           |                |            |                 |                          |                         |               | max. students    | 35              |  |
| Name of unit administrating study   |  |   | KMikr  |           | Field of study |            | Economics/MSG** |                          | Field of specialisation |               | NONE;            |                 |  |
| Teaching staff  |  |   | Elżbieta Babula, Ph.D.   |           |                |            |                 |                          |                         |               |                  |                 |  |
| Number of hours   |  |   |  |           |                |            |                 |                          |                         |               |                  |                 |  |
| Lectures  | 0  | Classes   | 30   | Tutorials | 0              | Laboratory |                 | 0                        | Seminars                | 0             | Language classes | 0               |  |
| Forma aktywności  |  |   |  |           |                |            |                 | Year&Type of studies*    |                         | 2 SS1, 1 SS2, |                  |                 |  |
| Hours with the participation of the academic teacher (including office hours, exams, others):   |  |   |  |           |                | 15         |                 | Semester:                |                         | 4, 2,         |                  |                 |  |
| Hours without the participation of the academic teacher (student's self-study, homeworks):  |  |   |  |           |                | 10         |                 | Type of course:          |                         | optional      |                  |                 |  |
| Total number of hours:  |  |   |  |           |                | 25         |                 | Language of instruction: |                         | English       |                  |                 |  |
| Teaching form   |  | in-class learning<br>Computer laboratory  |  |           |                |            |                 |                          |                         |               |                  |                 |  |
| Teaching methods  |  | Lectures including multimodal presentations, Activating methods in training classes, Work in computer laboratories, Individual projects,  |  |           |                |            |                 |                          |                         |               |                  |                 |  |
| Prerequisites (required courses and introductory requirements)  |  |   |  |           |                |            |                 |                          |                         |               |                  |                 |  |
| Required courses  |  | Completed Mathematical applications in economics and management course or other undergraduate mathematics course.   |  |           |                |            |                 |                          |                         |               |                  |                 |  |
| Introductory requirements   |  | The course requires basic knowledge of:<br>- differential calculus (derivatives and integrals of elementary functions with basic rules of differentiation and integration);<br>- matrix algebra.  |  |           |                |            |                 |                          |                         |               |                  |                 |  |
| Assessment method, forms and criteria   |  |   |  |           |                |            |                 |                          |                         |               |                  |                 |  |
| Assessment method   |  | Course completion (graded)  |  |           |                |            |                 |                          |                         |               |                  |                 |  |
| Assessment criteria   |  | To complete the course, the student has to accumulate at least 21 points. Student is awarded with points for:<br><br>- group work in-class activities (max 20 points),<br>- quizzes and tests online at fixed dates outside of class (max 10 points),<br>- assignments: two tasks to be solved individually outside of class within given deadline (max 20 points),<br>- high attendance (max 10 points). |  |           |                |            |                 |                          |                         |               |                  |                 |  |
| Course objectives   |  |   |  |           |                |            |                 |                          |                         |               |                  |                 |  |
| The purpose of this course is to help students develop advanced skills for formulating and analyzing mathematical models in the economics and finance. Rigorous mathematical analysis of theoretical models can lead to a better understanding of economic problems.<br>Additionally, the purpose is to help students develop skills for using the computer tools to solve mathematical models and to apply the knowledge in economic modeling. |  |   |  |           |                |            |                 |                          |                         |               |                  |                 |  |
| Learning outcomes   |  |   |  |           |                |            |                 |                          |                         |               |                  |                 |  |
| Knowledge   |  | MSG1_W10  | The student knows and understands the conditions for the existence of the solution of a linear system of equations; knows the types of constraint optimization problems and understands the solution methods; understands the qualitative solutions of basic dynamic problems; characterizes the dynamic programming and optimal control problems and knows its solution methods; understands the basic characteristics of Markov chains. The student knows the computer tools to support mathematical analysis. |           |                |            |                 |                          |                         |               |                  |                 |  |
|   |  | E1_W06  | The student knows and understands the conditions for the existence of the solution of a linear system of equations; knows the types of constraint optimization problems and understands the solution methods; understands the qualitative solutions of basic dynamic problems; characterizes the dynamic programming and optimal control problems and knows its solution methods; understands the basic characteristics of Markov chains. The  |           |                |            |                 |                          |                         |               |                  |                 |  |

|   |              |   |      |                        |                  |                         |                    |                      |                      |                    |               |
|---|--------------|---|------|------------------------|------------------|-------------------------|--------------------|----------------------|----------------------|--------------------|---------------|
|   |              | student knows the computer tools to support mathematical analysis.  |      |                        |                  |                         |                    |                      |                      |                    |               |
|   | E2_W06       | The student knows and understands the conditions for the existence of the solution of a linear system of equations; knows the types of constraint optimization problems and understands the solution methods; understands the qualitative solutions of basic dynamic problems; characterizes the dynamic programming and optimal control problems and knows its solution methods; understands the basic characteristics of Markov chains. The student knows the computer tools to support mathematical analysis.  |      |                        |                  |                         |                    |                      |                      |                    |               |
|   | MSG2_W13     | The student knows and understands the conditions for the existence of the solution of a linear system of equations; knows the types of constraint optimization problems and understands the solution methods; understands the qualitative solutions of basic dynamic problems; characterizes the dynamic programming and optimal control problems and knows its solution methods; understands the basic characteristics of Markov chains. The student knows the computer tools to support mathematical analysis.  |      |                        |                  |                         |                    |                      |                      |                    |               |
| Verification of learning outcomes - Knowledge |              |   |      |                        |                  |                         |                    |                      |                      |                    |               |
| Outcomes                                      | written exam | oral exam   | test | essay/paper /portfolio | tasks/ homeworks | individual presentation | group presentation | classroom activities | classroom discussion | individual project | group project |
| MSG1_W10                                      |              |   |      |                        | X                |                         |                    | X                    |                      | X                  |               |
| E1_W06  |              |   |      |                        | X                |                         |                    | X                    |                      | X                  |               |
| E2_W06  |              |   |      |                        | X                |                         |                    | X                    |                      | X                  |               |
| MSG2_W13                                      |              |   |      |                        | X                |                         |                    | X                    |                      | X                  |               |
| Skills  | MSG1_U02     | The student classifies the systems of linear equations and solves the systems if possible; identifies the basic types of differential equations and applies the correct solution method; solves constraint optimization problems and interprets the solution; analyzes and interprets the qualitative solutions of basic dynamic problems; solves and analyzes optimal control problems; applies matrix methods to discuss properties of Markov chains; can find the invariant measure of Markov chain. The student recognizes the areas of application for the methods in economics and finance. |      |                        |                  |                         |                    |                      |                      |                    |               |
|   | MSG1_U08     | The student applies the computer tools to solve problems that require mathematical methods.   |      |                        |                  |                         |                    |                      |                      |                    |               |
|   | MSG1_U14     | The student can cooperate in group to develop the solution for given task.  |      |                        |                  |                         |                    |                      |                      |                    |               |
|   | E1_U02       | The student classifies the systems of linear equations and solves the systems if possible; identifies the basic types of differential equations and applies the correct solution method; solves constraint optimization problems and interprets the solution; analyzes and interprets the qualitative solutions of basic dynamic problems; solves and analyzes optimal control problems; applies matrix methods to discuss properties of Markov chains; can find the invariant measure of Markov chain. The student recognizes the areas of application for the methods in economics and finance. |      |                        |                  |                         |                    |                      |                      |                    |               |
|   | E1_U04       | The student applies the computer tools to solve problems that require mathematical methods.   |      |                        |                  |                         |                    |                      |                      |                    |               |
|   | E1_U13       | The student can cooperate in group to develop the solution for given task.  |      |                        |                  |                         |                    |                      |                      |                    |               |
|   | E2_U02       | The student classifies the systems of linear equations and solves the systems if possible; identifies the basic types of differential equations and applies the correct solution method; solves constraint optimization problems and interprets the solution; analyzes and interprets the qualitative solutions of basic dynamic problems; solves and analyzes optimal control problems; applies matrix methods to discuss properties of Markov chains; can find the invariant measure of Markov chain. The student recognizes the areas of application for the methods in economics and finance. |      |                        |                  |                         |                    |                      |                      |                    |               |
|   | E2_U04       | The student applies the computer tools to solve problems that require mathematical methods.   |      |                        |                  |                         |                    |                      |                      |                    |               |
|   | E2_U13       | The student can cooperate in group to develop the solution for given task.  |      |                        |                  |                         |                    |                      |                      |                    |               |
|   | MSG2_U02     | The student classifies the systems of linear equations and solves the systems if possible; identifies the basic types of differential equations and applies the correct solution method; solves constraint optimization problems and interprets the solution; analyzes and interprets the qualitative solutions of basic dynamic problems; solves and analyzes optimal control problems; applies matrix methods to discuss properties of Markov chains;   |      |                        |                  |                         |                    |                      |                      |                    |               |

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|  |          | can find the invariant measure of Markov chain. The student recognizes the areas of application for the methods in economics and finance. |
|  | MSG2_U10 | The student applies the computer tools to solve problems that require mathematical methods.   |
|  | MSG2_U12 | The student can cooperate in group to develop the solution for given task.  |

**Verification of learning outcomes - Skills**

| Outcomes | written exam | oral exam | test | essay/paper /portfolio | tasks/ homeworks | individual presentation | group presentation | classroom activities | classroom discussion | individual project | group project |
|----------|--------------|-----------|------|------------------------|------------------|-------------------------|--------------------|----------------------|----------------------|--------------------|---------------|
| MSG1_U02 |              |           |      |                        | X                |                         |                    | X                    |                      | X                  |               |
| MSG1_U08 |              |           |      |                        | X                |                         |                    | X                    |                      | X                  |               |
| MSG1_U14 |              |           |      |                        |                  |                         |                    | X                    |                      |                    |               |
| E1_U02   |              |           |      |                        | X                |                         |                    | X                    |                      | X                  |               |
| E1_U04   |              |           |      |                        | X                |                         |                    | X                    |                      | X                  |               |
| E1_U13   |              |           |      |                        |                  |                         |                    | X                    |                      |                    |               |
| E2_U02   |              |           |      |                        | X                |                         |                    | X                    |                      | X                  |               |
| E2_U04   |              |           |      |                        | X                |                         |                    | X                    |                      | X                  |               |
| E2_U13   |              |           |      |                        |                  |                         |                    | X                    |                      |                    |               |
| MSG2_U02 |              |           |      |                        | X                |                         |                    | X                    |                      | X                  |               |
| MSG2_U10 |              |           |      |                        | X                |                         |                    | X                    |                      | X                  |               |
| MSG2_U12 |              |           |      |                        |                  |                         |                    | X                    |                      |                    |               |

|           |          |  |
|-----------|----------|--|
| Attitudes | MSG1_K02 | The student individually as well as in cooperation within group expands his or her awareness of possibilities and boundaries of applying mathematics to a better understanding of economic problems. |
|           | E1_K02   | The student individually as well as in cooperation within group expands his or her awareness of possibilities and boundaries of applying mathematics to a better understanding of economic problems. |
|           | E2_K02   | The student individually as well as in cooperation within group expands his or her awareness of possibilities and boundaries of applying mathematics to a better understanding of economic problems. |
|           | MSG2_K06 | The student individually as well as in cooperation within group expands his or her awareness of possibilities and boundaries of applying mathematics to a better understanding of economic problems. |

**Verification of learning outcomes - Attitudes**

| Outcomes | written exam | oral exam | test | essay/paper /portfolio | tasks/ homeworks | individual presentation | group presentation | classroom activities | classroom discussion | individual project | group project |
|----------|--------------|-----------|------|------------------------|------------------|-------------------------|--------------------|----------------------|----------------------|--------------------|---------------|
| MSG1_K02 |              |           |      |                        |                  |                         |                    | X                    |                      | X                  |               |
| E1_K02   |              |           |      |                        |                  |                         |                    | X                    |                      | X                  |               |
| E2_K02   |              |           |      |                        |                  |                         |                    | X                    |                      | X                  |               |
| MSG2_K06 |              |           |      |                        |                  |                         |                    | X                    |                      | X                  |               |

**Course contents**

1. Review of basic linear algebra: determinants and matrix inverses; Cramer's rule; rank of matrix; linear systems of equations; degrees of freedom. All tasks in this topic are conducted in a computer laboratory.
2. Linear programming: basic properties and examples of linear programs; basic solutions; the fundamental theorem of linear programming; the simplex method; dual linear programs. This topic is conducted with computer laboratory support.
3. Non-linear programming: constrained optimization with equality constraints (Lagrange problem) and with inequality



constraints (Kuhn-Tucker problem).

4. Differential equations: constant coefficient linear differential equations; qualitative solution: phase portrait diagrams; nonlinear systems; fixed points; linearization of dynamic system in the plane. This topic is conducted with computer laboratory support.

5. Difference equations: review of difference equations; linear difference equations; non-linear difference equations and phase diagram.

6. Optimal control: maximum principle; transversality conditions.

7. Dynamic programming: dynamic programming problems; the principle of optimality; the value function; Bellman equation.

8. Stochastic processes: Markov chains; stationary distributions. This topic is conducted with computer laboratory support.

#### Recommended reading lists

**Mandatory literature:**

K. Sydsater, P. Hammond, A. Seierstad, A. Strom, *Further mathematics for economic analysis*, Prentice Hall, 2005.

**Supplementary literature:**

1. Chiang A., *Elements of dynamic optimization*, McGraw-Hill 1992.
2. Chiang A., *Fundamental methods of mathematical economics*, McGraw-Hill 1967.
3. Brzeźniak Z., Zastawiak T., *Basic stochastic processes*, Springer 2003.

**Contact**

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\* SS1- undergraduate studies \* SS2 - graduate studies \* SDang - doctoral studies

\*\* MSG - International Economic Relations